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Gravitational particle production and the moduli problem

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Abstract: A theory of gravitational production of light scalar particles during and after inflation is investigated. We show that in the most interesting cases where long-wavelength fluctuations of light scalar fields can be generated during inflation, these fluctuations rather than quantum fluctuations produced after inflation give the dominant contribution to particle production. In such cases a simple analytical theory of particle production can be developed. Application of our results to the theory of quantum creation of moduli fields demonstrates that if the moduli mass is smaller than the Hubble constant then these fields are copiously produced during inflation. This gives rise to the cosmological moduli problem even if there is no homogeneous component of the classical moduli field in the universe. To avoid this version of the moduli problem it is necessary for the Hubble constant H during the last stages of inflation and/or the reheating temperature T_R after inflation to be extremely small.

Keywords: Nonperturbative Effects, Cosmology of Theories beyond the SM, Physics of the Early Universe.

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Contents

1. Introduction

Recently there has been a renewal of interest in gravitational production of particles in an expanding universe. This was a subject of intensive study many years ago, see e.g. [1]. However, with the invention of inflationary theory the issue of the production of particles due to gravitational effects became less urgent. Indeed, gravitational effects are especially important near the cosmological singularity, at the Planck time. But the density of the particles produced at that epoch becomes exponentially small due to inflation. New particles are produced only after the end of inflation when the energy density is much smaller than the Planck density. Production of particles due to gravitational effects at that stage is typically very inefficient.

There are a few exceptions to this rule that have motivated the recent interest in gravitational particle production. First of all, there are some models where the main mechanism of reheating during inflation is due to gravitational production. Even though this mechanism is very inefficient, in the absence of other mechanisms of reheating it may do the job. For example, one may consider the class of theories where the inflaton potential $V(\phi)$ gradually decreases at large ϕ and does not have any minima. In such theories the inflaton field ϕ does not oscillate after inflation, so the standard mechanism of reheating does not work [2, 3]. To emphasize this unusual feature of such theories we call them non-oscillatory models, or simply NO models [3]. Usually gravitational particle production in such models lead to dangerous cosmological consequences, such as large isocurvature fluctuations and overproduction of gravitinos [3]. In order to overcome these problems, it was necessary to modify the NO models [3] and to use the non-gravitational mechanism of instant preheating for the description of particle production [4].

There are some other cases where even very small but unavoidable gravitational particle production may lead either to useful or to catastrophic consequences [5, 6, 7, 8, 9]. For example, it has recently been found that the production of gravitinos by the oscillating inflaton field is not suppressed by the small gravitational coupling. As a result, gravitinos can be copiously produced in the early universe even if the reheating temperature always remains smaller than 10^8 GeV [8, 9]. Another important example is related to moduli production. 15 years ago Coughlan et al. realized that string theory and supergravity give rise to a cosmological moduli problem associated with the existence of a large homogeneous classical moduli field in the early universe [10]. Soon afterwards Goncharov, Linde and Vysotsky showed that quantum fluctuations of moduli fields produced at the last stages of inflation lead to the moduli problem even if initially there were no classical moduli fields [7]. Thus the cosmological moduli problem may appear either because of the existence of a large long-living homogeneous classical moduli field or because of quantum production of excitations (particles) of the moduli fields. In [3] it was pointed out that the problem of moduli production is especially difficult in the context of NO models, where moduli are produced as abundantly as usual particles.

Recently the problem of moduli production in the early universe was studied by numerical methods in [9], with conclusions similar to those of ref. [7]. As we are going to demonstrate, the main source of gravitational production of light moduli in inflationary cosmology is very simple, and one can study the theory of moduli production not only numerically but also analytically by the methods developed in [7, 3]. This will allow us to generalize and considerably strengthen the results of refs. [7, 9].

In particular, we will see that in the leading approximation the problem of overproduction of light moduli particles is *equivalent* to the problem of large homogeneous classical moduli fields [7]. We will show that the ratio of the number density of light moduli produced during inflation to the entropy of the universe after reheating satisfies the inequality

$$\frac{n_\chi}{s} \gtrsim \frac{T_R H_0^2}{3mM_p^2}\,. \tag{1.1}$$

Here m is the moduli mass, $M_p \sim 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and H_0 is the Hubble constant at the moment corresponding to the beginning of the last 60 e-foldings before the end of inflation.

In the simplest versions of inflationary theory with potentials $M^2\phi^2/2$ or $\lambda\phi^4/4$ one has $H_0 \sim 10^{14}$ GeV. In such models our result implies that in order to satisfy the cosmological constraint $n_\chi/s \lesssim 10^{-12}$ one needs to have an abnormally small reheating temperature $T_R \lesssim 1$ GeV. Alternatively one may consider inflationary models where the Hubble constant at the end of inflation is very small. But we will argue that even this may not help, so one may need either to invoke thermal inflation or to use some other mechanisms which can make the moduli problem less severe, see e.g. [11, 12].

In the next section we outline the classical and quantum versions of the moduli problem and explain how each of them can arise in inflationary theory. In section 3 we describe the results of our numerical simulations of gravitational production of light scalar fields during and after preheating. In particular we verify our prediction that the dominant contribution to particle production comes from long-wavelength modes which are indistinguishable from homogeneous classical moduli fields. Finally in section 4 we analytically compute the production of these long wavelength modes and derive eq. (1.1). This section also contains our concluding discussion.

2. Moduli problem

String moduli couple to standard model fields only through Planck scale suppressed interactions. Their effective potential is exactly flat in perturbation theory in the supersymmetric limit, but it may become curved due to nonperturbative effects or because of supersymmetry breaking. If these fields originally are far from the minimum of their effective potential, the energy of their oscillations will decrease in an expanding universe in the same way as the energy density of nonrelativistic matter, $\rho_m \sim a^{-3}(t)$. Meanwhile the energy density of relativistic plasma decreases as a^{-4}. Therefore the relative contribution of moduli to the energy density of the universe may quickly become very significant. They are expected to decay after the stage of nucleosynthesis, violating the standard nucleosynthesis predictions unless the initial amplitude of the moduli oscillations χ_0 is sufficiently small. The constraints on the energy density of the moduli field ρ_χ and the number of moduli particles n_χ depend on details of the theory. The most stringent constraint appears because of the photodissociation and photoproduction of light elements by the decay products of the moduli fields. For $m \sim 10^2 - 10^3$ GeV one has

$$\frac{n_\chi}{s} \lesssim 10^{-12} - 10^{-15} \,. \tag{2.1}$$

see [13, 14] and references therein. In this paper we will use a conservative version of this constraint, $n_\chi/s \lesssim 10^{-12}$.

If the field χ is a classical homogeneous oscillating scalar field, then this constraint applies to it as well if one defines the corresponding number density of nonrelativistic particles χ by the following obvious relation:

$$n_\chi = \frac{\rho_\chi}{m} = \frac{m\chi^2}{2} \,. \tag{2.2}$$

Let us first consider moduli χ with a constant mass $m \sim 10^2 - 10^3$ GeV and assume that reheating of the universe occurs after the beginning of oscillations of the moduli. This is indeed the case if one takes into account that in order to avoid the gravitino problem one should have $T_R < 10^8$ GeV. We will also assume for definiteness that the minimum of the effective potential for the field χ is at $\chi = 0$; one can always achieve this by an obvious redefinition of the field χ.

Independent of the choice of inflationary theory, at the end of inflation the main fraction of the energy density of the universe is concentrated in the energy of an oscillating scalar field ϕ. Typically this is the same field which drives inflation, but in some models such as hybrid inflation this may not be the case. We will consider here the simplest (and most general) model where the effective potential of the field ϕ after inflation is quadratic,

$$V(\phi) = \frac{M^2}{2}\phi^2 . \tag{2.3}$$

After inflation the field ϕ oscillates. If one keeps the notation ϕ for the amplitude of oscillations of this field, then one can say that the energy density of this field is given by $\rho(\phi) = \frac{M^2}{2}\phi^2$.

To simplify our notation, we will take the scale factor at the end of inflation to be $a_0 = 1$. The amplitude of the oscillating field in the theory with the potential (2.3) changes as

$$\phi(t) = \phi_0 \; a^{-3/2}(t) . \tag{2.4}$$

The field χ does not oscillate and almost does not change its magnitude until the moment t_1 when $H^2(t) = \rho_\phi/3M_p^2$ becomes smaller than $m^2/3$. At that time one has

$$\frac{\rho_\chi}{\rho_\phi} \sim \frac{m^2\chi_0^2}{6H^2(t)M_p^2} \sim \frac{\chi_0^2}{2M_p^2} . \tag{2.5}$$

This ratio, which can also be obtained by a numerical investigation of oscillations of the moduli fields, does not change until the time t_R when reheating occurs because ρ_χ and ρ_ϕ decrease in the same way: they are proportional to a^{-3}.

At the moment of reheating one has $\rho_\phi(t_R) = \pi^2 N(T) T_R^4/30$, and the entropy of produced particles $s = 2\pi^2 N(T) T_R^3/45$, where $N(T)$ is the number of light degrees of freedom. This yields

$$\frac{n_\chi}{s} \sim \frac{\rho_\chi}{ms} \sim \frac{\chi_0^2 T_R}{3mM_p^2} . \tag{2.6}$$

Usually one expects $T_R \gg m \sim 10^2$ GeV. Then in order to have $n_\chi/s < 10^{-12}$ one would need $\chi_0 \ll 10^{-6} M_p$. However, it is hard to imagine why the value of the moduli field at the end of inflation should be so small. If one takes $\chi_0 \sim M_p$, which looks natural, then one violates the bound $n_\chi/s < 10^{-12}$ by more than 12 orders of magnitude. This is the essence of the cosmological moduli problem [10].

In general, the situation is more complex. During the expansion of the universe the effective potential of the moduli acquires some corrections. In particular, quite often the effective mass of the moduli (the second derivative of the effective potential) can be represented as

$$m_\chi^2 = m^2 + c^2 H^2 , \tag{2.7}$$

where c is some constant and H is the Hubble parameter [15]. Higher derivatives of the effective potential may acquire corrections as well. This leads to a different version of the moduli problem discussed in [7], see also [16]. The position of the minimum of the effective potential of the moduli field in the early universe may occur at a large distance from the position of the minimum at present. This may fix the initial position of the field χ and lead to its subsequent oscillations.

A simple toy model illustrating this possibility was given in [11]:

$$V = \frac{1}{2} m_\chi^2 \chi^2 + \frac{c^2}{2} H^2 \left(\chi - \chi_0\right)^2 . \tag{2.8}$$

At large H the minimum appears at $\chi = \chi_0$; at small H the minimum is at $\chi = 0$. Thus one would expect that initially the field should stay at χ_0, and later, when H decreases, it should oscillate about $\chi = 0$ with an initial amplitude approximately equal to χ_0. The only natural value for χ_0 in supergravity is $\chi_0 \sim M_p$. This may lead to a strong violation of the bound (2.1).

A more detailed investigation of this situation has shown [12] that one should distinguish between three different possibilities: $c \gg 1$, $c \sim 1$ and $c \ll 1$.

If $c > O(10)$, the field χ is trapped in the (moving) minimum of the effective potential, its oscillations have very small amplitudes, and the moduli problem does not appear at all [12]. This is the simplest resolution of the problem, but it is not simple to find realistic models where the condition $c > O(10)$ is satisfied.

The most natural case is $c \sim 1$. It requires a complete study of the behavior of the effective potential in an expanding universe. There may exist some cases where the minimum of the effective potential does not move in this regime, but in general the effects of quantum creation of moduli in this scenario [7, 9] are subdominant with respect to the classical moduli problem discussed above [7, 16], so we will not discuss this regime in our paper.

Here we will study the case $c \ll 1$. In this case the effective mass of the moduli at $H \gg m$ is always much smaller than H, so the field does not move towards its minimum, regardless of its position. Thus if there is any classical field χ_0 it simply stays at its initial position until H becomes smaller than m, just as in the case considered above, and the resulting ratio n_χ/s is given by eq. (2.6).

The moduli problem in this scenario has two aspects. First of all, in order to avoid the classical moduli problem one needs to explain why $\chi_0 \sim 10^{-6}\sqrt{m/T_R}\, M_p$, which is necessary (but not sufficient) to have $n_\chi/s < 10^{-12}$. Then one should study quantum creation of moduli in an expanding universe and check whether their contribution to n_χ violates the bound $n_\chi/s < 10^{-12}$. This last aspect of the moduli problem was studied in [7, 9].

In inflationary cosmology these two contributions (the contributions to n_χ from the classical field χ and from its quantum fluctuations) are almost indistinguishable. Indeed, the dominant contribution to the number of moduli produced in an expanding universe is given by the fluctuations of the moduli field produced during inflation. These fluctuations have exponentially large wavelengths and for all practical purposes they have the same consequences as a homogeneous classical field of amplitude $\chi_0 = \sqrt{\langle\chi^2\rangle}$.

To be more accurate, these fluctuations behave in the same way as the homogeneous classical field ϕ only if their wavelength is greater than H^{-1}. During inflation this condition is satisfied for all inflationary fluctuations, but after inflation the size of the horizon grows and eventually becomes larger than the wavelength of some of the modes. Then these modes begin to oscillate and their amplitude begins to decrease even if at that stage $m < H$. To take this effect into account one may simply exclude from consideration those modes whose wavelengths become smaller than H^{-1} prior to the moment $t \sim m^{-1}$ when H drops down to m. It can be shown that in the context of our problem this is a relatively minor correction, so we can use the simple estimate $\chi_0 = \sqrt{\langle\chi^2\rangle}$.

In order to evaluate this quantity we will assume that $c \ll 1$ and $m \ll H$ during and after inflation. This reduces the problem to the investigation of the production of massless (or nearly massless) particles during and after inflation. In the next section we study this issue and show that in the most interesting cases where inflationary long-wavelength fluctuations of a scalar field can be generated during inflation, they give the dominant contribution to particle production. This allows us to reduce a complicated problem of gravitational particle production to a simple problem which can be easily solved analytically.

3. Generation of light particles from and after inflation

In this section we will present the results of a numerical study of the gravitational creation of light scalar particles in the context of inflation. Consider a scalar field χ with the potential

$$V(\chi) = \frac{1}{2}\left(m^2 - \xi R\right)\chi^2\,, \tag{3.1}$$

where R is the Ricci scalar. In a Friedmann universe $R = -\frac{6}{a^2}(\ddot{a}a + \dot{a}^2)$. The scalar field operator can be represented in the form

$$\chi(x,t) = \frac{1}{(2\pi)^{-3/2}}\int d^3k\left[\hat{a}_k\chi_k(t)e^{ikx} + \hat{a}_k^\dagger\chi_k^*(t)e^{-ikx}\right], \tag{3.2}$$

where the eigenmode functions χ_k satisfy

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left[\left(\frac{k}{a}\right)^2 + m^2 - \xi R\right]\chi_k = 0\,. \tag{3.3}$$

By introducing conformal time and field variables defined as $\eta \equiv \int dt/a$, $f_k \equiv a\chi_k$ eq. (3.3) can be simplified to

$$f_k'' + \omega_k^2 f_k = 0\,, \tag{3.4}$$

where primes denote differentiation with respect to η and

$$\omega_k^2 = k^2 + a^2 m^2 + \left(\frac{1}{6} - \xi\right) \frac{a''}{a}\,. \tag{3.5}$$

The growth of the scale factor is determined by the evolution of the inflaton field ϕ with potential $V(\phi)$. In conformal time

$$\begin{aligned} a'' &= \frac{a'^2}{a} - \frac{8\pi a}{3}\left(\phi'^2 - a^2 V(\phi)\right) \\ \phi'' &+ 2\frac{a'}{a}\phi' + a^2 \frac{\partial V(\phi)}{\partial \phi} = 0\,. \end{aligned} \tag{3.6}$$

For initial conditions for the modes f_k, in the first approximation one can use positive frequency vacuum fluctuations $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$, see e.g. [6]. However, when describing fluctuations produced at the last stages of a long inflationary period, one should begin with fluctuations which have been generated during the previous stage of inflation. For example, for massless scalar fields minimally coupled to gravity instead of $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$ one should use Hankel functions [17]:

$$f_k(t) = \frac{ia(t)\,\mathrm{H}}{k\sqrt{2\,k}} \left(1 + \frac{k}{i\,\mathrm{H}} e^{-\mathrm{H}\,t}\right) \exp\left(\frac{i\,k}{\mathrm{H}}\, e^{-\mathrm{H}\,t}\right), \tag{3.7}$$

where H is the Hubble constant at the beginning of calculations. To make the calculations even more accurate, one should take into account that long-wavelength perturbations were produced at early stages of inflation when H was greater than at the beginning of the calculations. If the stage of inflation is very long, then the final results do not change much if instead of the Hankel functions (3.7) one uses $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$. However, if the inflationary stage is short, then using the functions $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$ considerably underestimates the resulting value of $\langle \chi^2 \rangle$.

At late times the solutions to eq. (3.4) can be represented in terms of WKB solutions as

$$f_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k}} e^{-i\int^\eta \omega_k d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k}} e^{+i\int^\eta \omega_k d\eta}\,, \tag{3.8}$$

where $\alpha_k(\eta)$ and $\beta_k(\eta)$ play the role of coefficients of a Bogolyubov transformation. This form is often used to discuss particle production because the number density of particles in a given mode is given by $n_k = |\beta_k(\eta)|^2$ and their energy density is $\omega_k n_k$. As we will see, though, the main contribution to the number density of χ particles at late times comes from long-wavelength modes which are far outside the horizon during reheating. As long as they remain outside the horizon these modes do not manifest particle-like behaviour, i.e. the mode functions do not oscillate. In this situation the coefficients α and β have no clear physical meaning. We therefore present our results in terms of the mode amplitudes $|f_k(\eta)|^2$, which as we will show contain all the information relevant to number density and energy density at late times.

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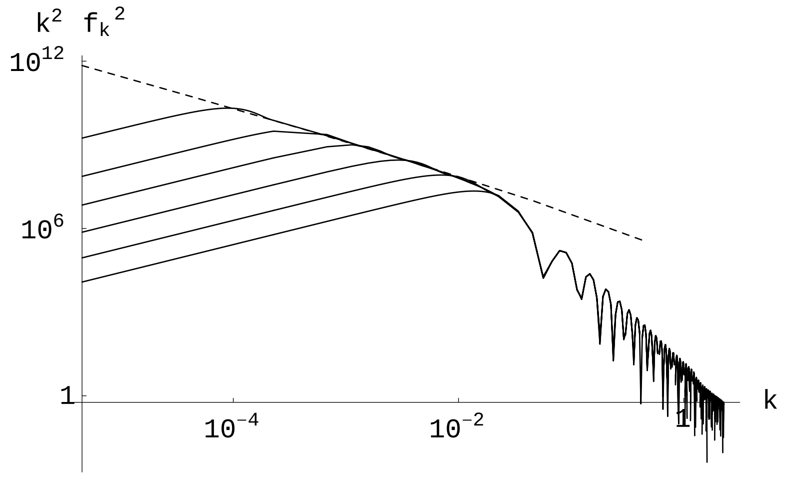

Figure 1: Fluctuations vs. mode frequency at a late time. The lower plots show runs which started close to the end of inflation, with initial conditions $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$. The starting times range from $\phi_0 = 1.5M_p$ (lowest curve) up through $\phi_0 = 2M_p$. The highest curve shows the Hankel function solutions given by eqs. (3.10) and (3.11) As we see, calculations with $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$ produce the correct spectrum at large k but underestimate the level of quantum fluctuations at small k.

At late times when $a''/a \sim H < m$ the long wavelength modes of χ will be nonrelativistic and their number density will simply be given by eq. (2.2). Moreover the very long wavelength modes which are still outside the horizon at late times (e.g. at nucleosynthesis) will act like a classical homogeneous field whose amplitude is given by

$$\langle \chi^2 \rangle = \frac{1}{2\pi^2 a^2} \int dk k^2 |f_k|^2 \,. \tag{3.9}$$

It is these very long wavelength modes which will dominate and therefore the quantity of interest for us is the amplitude of these fluctuations.

In our calculations we assumed that $m^2 = c^2 H^2$ with $c \ll 1$; the results shown are for $c = 0$ but we also did the calculations with $c = .01$ and found that the results were independent of c in this range.

Figure 1 shows the results of solving eq. (3.4) for a model with the inflaton potential $V(\phi) = \frac{1}{4}\lambda\phi^4$. These data were taken after ten oscillations of the inflaton field. The vertical axis shows $k^2|f_k|^2$ as a function of the momentum k. The momentum is shown in units of the Hubble constant at the end of inflation.

The different plots represent runs with different starts of inflation, i.e. with different initial values of ϕ. They all coincide in the ultraviolet part of the spectrum, but the runs which started towards the end of inflation show a significant suppression in the infrared. This shows that fluctuations produced during inflation are the primary source of the infrared modes, which in turn dominate the number density.

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The curve on top shows the Hankel function solutions (3.7), which give

$$|f_k|^2 = \frac{a^2 H^2}{2k^3} \tag{3.10}$$

for de Sitter space, i.e. for a constant H. In the figure, we have corrected this expression by using for each mode the value of the Hubble constant at the moment when that mode crossed the horizon. For the $\lambda\phi^4$ model shown here the appropriate Hubble parameter for each mode can be approximated as

$$H_k = \sqrt{\frac{2\pi\lambda}{3}} \left(\phi_e^2 - \frac{1}{\pi} \ln\left(\frac{k}{H_e}\right)\right), \tag{3.11}$$

where ϕ_e and H_e are the values of the inflaton and the Hubble parameter respectively at the end of inflation.

Note that if the Hankel function solutions (3.7) are used as initial conditions for a numerical run then they do not change as the modes cross the horizon, so the upper curve of the plot can also be obtained from such a run. Relative to this upper curve it's easy to see how the numerical runs show suppression in the infrared due to starting inflation at late times and choosing the initial conditions in the form $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$, and suppression in the ultraviolet due to the end of inflation. The latter suppression is physically realistic. The infrared suppression should occur at a wavelength corresponding to the Hubble radius at the beginning of inflation.

The different regions of the graph illustrate effects which occurred at different times. During inflation long wavelength modes crossed the horizon at early times. Thus the far left portion of the plot shows the modes which crossed earliest. They have the highest amplitudes both because they were frozen in earliest and because the Hubble constant was higher at earlier times when they were produced. The lower plots don't show these high amplitudes because inflation began too late for these modes to cross outside the horizon and be amplified. Farther to the right the curve shows modes which were only slightly if at all amplified during inflation. The far right modes were produced during the fast rolling and oscillatory stages. These modes are not frozen and can be described meaningfully in terms of α and β coefficients. The regularized expression

$$|f_k|^2 = \frac{1}{\omega_k} \left[|\beta_k|^2 + Re\left(\alpha_k \beta_k^* e^{-2i\int \omega_k d\eta}\right)\right] \tag{3.12}$$

shows why the amplitudes of these modes oscillate as a function of k.

In short inflationary fluctuations are primarily responsible for producing infrared modes and post-inflationary effects account for ultraviolet modes, but it is the infrared modes that were outside the horizon at the end of inflation which dominate the number density at late times. The earlier inflation began the farther this distribution will extend into the infrared, and the long wavelength end of this spectrum will always give the greatest contribution to the number density of χ particles.

Our numerical calculations are similar to those of Kuzmin and Tkachev [6]. However, they took a rather small initial value of the classical scalar field ϕ, which resulted in less than 60 e-folds of inflation. As initial conditions for the fluctuations they used $f_k = \frac{1}{\sqrt{2k}} e^{-ikt}$. They pointed out that the results of such calculations can give only a lower bound on the number of χ particles produced during inflation. Consequently, similar calculations performed in [9] could give only a lower bound on the number of moduli fields produced in the early universe.

Our goal is to find not a lower bound but the complete number of particles produced at the last stages of inflation in realistic inflationary models, where the total duration of inflation typically is much greater than 60 e-foldings. One result revealed by our calculations is that the effects of an arbitrarily long stage of inflation can be mimicked by the correct choice of "initial" conditions chosen for the modes χ_k after inflation. Instead of using the Minkowski space fluctuations $f_k = \frac{1}{\sqrt{k}} e^{-ikt}$ used in [6] as well as in our numerical calculations, one should use the de Sitter space solutions (3.7), with H corrected to the value it had for each mode at horizon crossing. Using these modes at the end of inflation is equivalent to running a simulation with a long stage of inflation.

Our numerical calculations confirmed the result that we are going to derive analytically in the next section (see also [3]): The number of χ particles ($n_\chi \sim m\langle\chi^2\rangle$) produced during the stage of inflation beginning at $\phi = \phi_0$ in the simplest model $M^2\phi^2/2$ is proportional to ϕ_0^4, whereas in the model $\lambda\phi^4/4$ it is proportional to ϕ_0^6. Thus the total number of particles produced during inflation is *extremely* sensitive to the choice of initial conditions. If one considers ϕ_0 corresponding to the beginning of the last 60 e-folds of inflation, the total number of particles produced at that stage appears to be much greater than the lower bound obtained in [6]. As we will see, this will allow us to put a much stronger constraint on the moduli theories than the constraint obtained in [9].

4. Light moduli from inflation

The numerical results obtained in the previous section confirm our expectation that in the most interesting cases where long-wavelength inflationary fluctuations of light scalar fields can be generated during inflation, they give the dominant contribution to particle production. In particular, in the case of $c \ll 1$, $m \ll H$ most moduli field fluctuations are generated during inflation rather than during the post-inflationary stage. These fluctuations grow at the stage of inflation in the same way as if the moduli field χ were massless [17]:

$$\frac{d\langle\chi^2\rangle}{dt} = \frac{H^3}{4\pi^2}\,. \tag{4.1}$$

If the Hubble constant does not change during inflation, one obtains the well-known relation

$$\langle\chi^2\rangle = \frac{H^3\, t}{4\pi^2}\,. \tag{4.2}$$

However, in realistic inflationary models the Hubble constant changes in time, and fluctuations of the light fields χ with $m \ll H$ behave in a more complicated way.

As an example, let us consider the case studied in the last section. Here inflation is driven by a field ϕ with an effective potential $V(\phi) = \frac{\lambda}{4}\phi^4$ at $\phi > 0$. This potential could be oscillatory or flat for $\phi < 0$. We consider a light scalar field χ which is not coupled to the inflaton field ϕ, and which is minimally coupled to gravity.

The field ϕ during inflation obeys the following equation:

$$3H\dot{\phi} = -\lambda\phi^3\,. \tag{4.3}$$

Here

$$H = \frac{1}{2}\sqrt{\frac{\lambda}{3}}\,\frac{\phi^2}{M_p}\,. \tag{4.4}$$

These two equations yield the solution [17]

$$\phi = \phi_0\ \exp\left(-2\sqrt{\frac{\lambda}{3}}M_p t\right), \tag{4.5}$$

where ϕ_0 is the initial value of the inflaton field ϕ. In this case eq. (4.1) reads:

$$\frac{d\langle\chi^2\rangle}{dt} = \frac{\lambda\sqrt{\lambda}}{96\sqrt{3}\pi^2}\frac{\phi_0^6}{M_p^3}\exp\left(-12\sqrt{\frac{\lambda}{3}}M_p t\right) \tag{4.6}$$

The result of integration at large t converges to

$$\langle\chi^2\rangle = \frac{\lambda}{2}\left(\frac{\phi_0^3}{24\pi M_p^2}\right)^2\,. \tag{4.7}$$

This result agrees with the results of our numerical investigation described in the previous section.

From the point of view of the moduli problem, these fluctuations lead to the same consequences as a classical scalar field χ which is homogeneous on the scale H^{-1} and which has a typical amplitude

$$\chi_0 = \sqrt{\langle\chi^2\rangle} = \sqrt{\frac{\lambda}{2}}\,\frac{\phi_0^3}{24\pi M_p^2}\,. \tag{4.8}$$

A similar result can be obtained in the model $V(\phi) = \frac{M^2}{2}\phi^2$. In this case one has [17]

$$\phi(t) = \phi_0 - \sqrt{\frac{2}{3}}\,M_p M t\,. \tag{4.9}$$

The time-dependent Hubble parameter is given by

$$H = \frac{M}{\sqrt{6}M_p}\phi(t)\,, \tag{4.10}$$

which yields

$$\chi_0 = \sqrt{\langle\chi^2\rangle} = \frac{M\phi_0^2}{8\pi\sqrt{3}M_p^2}\,. \tag{4.11}$$

As we see, the value of χ_0 depends on the initial value of the field ϕ. This result has the following interpretation. One may consider an inflationary domain of initial size $H^{-1}(\phi_0)$. This domain after inflation becomes exponentially large. For example, its size in the model with $V(\phi) = \frac{M^2}{2}\phi^2$ becomes [17]

$$l \sim H^{-1}(\phi_0)\exp\left(\frac{\phi_0^2}{4M_p^2}\right). \tag{4.12}$$

In order to achieve 60 e-folds of inflation in this model one needs to take $\phi_0 \sim 15M_p$. This implies that a typical value of the (nearly) homogeneous scalar field χ in a universe which experienced 60 e-folds of inflation in this model is given by

$$\chi_0 = \sqrt{\langle\chi^2\rangle} \sim 5M\,. \tag{4.13}$$

In realistic versions of this model one has $M \sim 5\times 10^{-6}M_p \sim 10^{13}$ GeV [17]. Substitution of this result into eq. (2.6) gives

$$\frac{n_\chi}{s} \sim 2\times 10^{-10}\,\frac{T_R}{m}\,. \tag{4.14}$$

This implies that the condition $n_\chi/s \lesssim 10^{-12}$ requires that the reheating temperature in this model should be at least two orders of magnitude smaller than m. For example, for $m \sim 10^2$ GeV one should have $T_R \lesssim 1$ GeV, which looks rather problematic.

This result confirms the basic conclusion of ref. [7] that the usual models of inflation do not solve the moduli problem. Our result is similar to the result obtained in [9] by numerical methods, but it is approximately two orders of magnitude stronger. The reason for this difference is that the authors of ref. [9] used a much smaller value of ϕ_0 in their numerical calculations. Consequently, they took into account only the particles produced at the very end of inflation, whereas the leading effect occurs at earlier stages of inflation, i.e. at larger ϕ.

In general one can get a simple estimate of $\chi_0 = \sqrt{\langle\chi^2\rangle}$ by assuming that the universe expanded with a constant Hubble parameter H_0 during the last 60 e-folds of inflation. To make this estimate more accurate one should take the value of the Hubble constant not at the end of inflation but approximately 60 e-foldings before it, at the time when the fluctuations on the scale of the present horizon were produced. The reason is that the largest contribution to the fluctuations is given by the time

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when the Hubble constant took its greatest values. Also, at that stage the rate of change of H was relatively slow, so the approximation $H = H_0 = \text{const.}$ is reasonable. Thus one can write

$$\chi_0 = \sqrt{\langle\chi^2\rangle} \gtrsim \frac{H_0}{2\pi}\sqrt{H_0 t} \sim \frac{H_0}{2\pi}\sqrt{60} \sim H_0\,. \tag{4.15}$$

This gives

$$\frac{n_\chi}{s} \gtrsim \frac{T_R\, H_0^2}{3m\, M_p^2}\,. \tag{4.16}$$

In the simplest versions of chaotic inflation with potentials $M^2\phi^2/2$ or $\lambda\phi^4/4$ one has $H_0 \sim 10^{14}\,\text{GeV}$, which leads to the requirement $T_R \lesssim 1\,\text{GeV}$. But this equation shows that there is another way to relax the problem of the gravitational moduli production: one may consider models of inflation with a very small value of H_0 [7]. For example, one may have $n_\chi/s \sim 10^{-12}$ for $T_R \sim H_0 \sim 10^7\,\text{GeV}$.

However, this condition is not sufficient to resolve the moduli problem; the situation is more complicated. First of all, it is very difficult to find inflationary models where inflation occurs only during 60 e-foldings. Typically it lasts much longer, and the fluctuations of the light moduli fields will be much greater. This is especially obvious in the theory of eternal inflation where the amplitude of fluctuations of the light moduli fields can become indefinitely large [17]. In particular, if the condition $m_\chi^2 \sim m^2 + c^2H^2$ with $c \ll 1$ remains valid for $\chi \gtrsim M_p$, then one may expect the generation of moduli fields $\chi > M_p$. This should initiate inflation driven by the light moduli [3]. Then the situation would become even more problematic: we would need to find out how one could produce baryon asymmetry of the universe after the light moduli decay and how one could obtain density perturbations $\delta\rho/\rho \sim 10^{-4}$ in such a low-scale inflationary model.

One may expect that the region of values of χ where its effective potential has small curvature $m^2 \ll H^2$ may be limited, and may even depend on H. Then the existence of a long stage of inflation would push the fluctuations of the field χ up to the region where its effective potential becomes curved, and instead of our results for χ_0 one should substitute the largest value of χ for which $m_\chi^2 < H^2$. In such a situation one would have a mixture of problems which occur at $c \ll 1$ and at $c \sim 1$.

Finally, we should emphasize that all our worries about quantum creation of moduli appear only after one makes the assumption that for whatever reason the initial value of the classical field χ in the universe vanishes, i.e. that the classical version of the moduli problem has been resolved. We do not see any justification for this assumption in theories where the mass of the moduli field in the early universe is much smaller than H. Indeed, in such theories the classical field χ initially can take *any* value, and this value is not going to change until the moment $t \sim H^{-1} \sim m^{-1}$. The main purpose of this paper was to demonstrate that even if one finds a solution to the light moduli problem at the classical level, the same problem will appear again because of quantum effects.

This does not mean that the moduli problem is unsolvable. One of the most interesting solutions is provided by thermal inflation [11]. The Hubble constant during inflation in this scenario is very small, and the effects of moduli production there are rather insignificant. Another possibility is that moduli are very heavy in the early universe, $m_\chi^2 = m^2 + c^2 H^2$, with $c > O(10)$, in which case the moduli problem does not appear [12]. The main question is whether we really need to make the theory so complicated in order to avoid the cosmological problems associated with moduli. Is it possible to find a simpler solution? One of the main results of our investigation is to confirm the conclusion of ref. [7] that the simplest versions of inflationary theory do not help us to solve the moduli problem but rather aggravate it.

In conclusion we would like to note that the methods developed in this paper apply not only to the theory of moduli production but to other problems as well. For example, one may study the theory of gravitational production of superheavy scalar particles after inflation [5, 6]. If these particles are minimally coupled to gravity and have mass $m \ll H$ during inflation, then one can use our eqs. (4.8), (4.11) to calculate the number of produced particles. These equations imply that the final result will strongly depend on ϕ_0, i.e. on the duration of inflation. If inflation occurs for more than 60 Hubble times, the production of particles with $m \ll H$ is much more efficient than was previously anticipated. As we just mentioned, if ϕ_0 is large enough then the production of fluctuations of the field χ may even lead to a new stage of inflation driven by the field χ [3]. On the other hand, if m is greater than the value of the Hubble constant at the very end of inflation, then quantum fluctuations are produced only at the early stages of inflation (when $H > m$). These fluctuations oscillate and decrease exponentially during the last stages of inflation. In such cases the final number of produced particles will not depend on the duration of inflation and can be unambiguously calculated. We hope to return to this question in a separate publication.

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